Probing Wigner Crystals in the 2DEG using Microwaves

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Based on work from the groups of:
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The Wigner Crystal

- In all electron gasses, there is a competition between the coulomb repulsion and the quantum delocalization energy:

  \[ E_{\text{coul}} \sim \frac{e^2}{l} \sim \sqrt{n} \quad \quad E_{\text{deloc}} \sim \frac{\hbar^2}{2ml^2} \sim n \]

- At low densities, the coulomb interaction will dominate, and electrons will crystallize into a hexagonal lattice in 2D.
Magnetic Field Enhancement of the WC

- In a magnetic field, electrons are not in plane wave states, but are instead localized by the magnetic field to a length $l_B = \sqrt{\hbar/eB}$.
- In this case, electrons will crystallize when the magnetic length becomes small compared to their spatial separation.
- This allows you to coerce electrons into crystal without needing to go to such low densities.
Wigner Crystals in Real Life

- The first real-life example of a Wigner Crystal was discovered in electrons on the surface of Liquid helium in 1979 (classical Wigner Crystal), Grimes and Adams, PRL 42 795 (1979)

- At very high fields, below the 1/5 fractional Hall state, the 2DEG becomes an insulator, and non-linear I-V characteristics (PRL 66 3285 (1991)) suggest that it may be a pinned Wigner Crystal.
Why measure with Microwaves?

- What happens to electrons at these high frequencies? Is there any structure or is it just mush?

- Can we gain information about the Wigner Crystal by probing it at high frequencies?

- The DC conductivity of the Quantum Hall effect has provided, and continues to provide, a wealth of information about the nature of electrons in 2D in magnetic fields. Can we get new information by going to microwave frequencies?
Experimental Setup

- A Coplanar Waveguide (CPW) is patterned on top of the sample.
- The relative width of the center conductor to the width of the gap to the ground planes is kept constant so that the transmission line has an impedance of 50 ohms.
Experimental Setup (cont’d)

- Microwaves are generated at Room Temperature and transmitted down to the waveguide.

- A diode detector is attached to the output of the waveguide to measure the transmitted power.

- A feedback loop maintains $V_0$ at a constant value.

![Diagram of experimental setup]
Relating Transmitted Power to $\sigma_{xx}$

- Waveguide couples capacitively to the 2DEG.

- Since $d = 100 \text{ nm} \ll w = 30 \mu \text{m}$, the fields through the 2DEG will be nearly the same magnitude as the fields between the waveguide electrodes.

- Since $\lambda \gg w$, we can also use the DC field distribution (quasi-TEM approximation): $\phi(x, t) = \phi_{DC}(x)e^{iwt}$
Relating $P$ to $\sigma_{xx}$ (cont’d)

- Effect of $\sigma_{xx}$ of 2DEG is to damp the wave as it travels down the waveguide.

- The experiment provides a direct measure of $\sigma_{xx}$ at a continuously adjustable microwave frequency by measuring the transmitted power through the waveguide.

- This transmitted power is related to $\sigma_{xx}$ by:

$$ P = \exp \left( - \frac{\sigma_{xx} Z_0 d}{w} \right) $$
Derivation of formula for $P$

Power dissipated by the electron in the 2DEG per unit length of the waveguide will be $P_{diss} = V^2 / R$, where $R$ is the resistance of the path of the electrons through the 2DEG.

For the current distribution from the AC field, we will have:

$$R = \frac{w}{\sigma_{xx}}$$

\[\begin{array}{c}
\hline
\text{\small $w$} \\
\hline
\end{array}\]
Derivation of formula for $P$ (cont’d)

This then gives

$$P_{diss} = \frac{\sigma_{xx} V^2}{w} = \frac{\sigma_{xx} Z_0 P}{w}$$

for the power dissipated per unit length. We then have:

$$P(l + \Delta l) = P(l) - \frac{\sigma_{xx} Z_0 P(l)}{w} \Delta l$$

$$\frac{dP}{dl} = -\frac{\sigma_{xx} Z_0}{w} P(l)$$

$$P(l) = P_0 \exp \left( -\frac{\sigma_{xx} Z_0 l}{w} \right)$$
Why did we ignore the 2DEG directly underneath the center conductor?

- The capacitance from the center conductor to the 2DEG is very large due to the close proximity of the 2DEG.
- As long as the resistivity of the 2DEG is large, at high frequencies, we will not be able to charge up the capacitor.
- The charging time will be at least: \( \tau = \left( \frac{w}{\sigma_{xx}} \right) \cdot (C_c w) \) where \( C_c \) is the capacitance per unit area from the top metal to the 2DEG. For \( 1/\sigma_{xx} \) of 10 k\( \Omega \), and \( d = 100 \text{ nm} \), this gives a frequency of 100 MHz.
Probing the High Field Insulator with Microwaves

2DHS, \( n = 5 \) to \( 1 \times 10^{10} \), \( \mu = 2.5 \times 10^5 \), PRB 61 10905 (2000)

- At very large fields, \( \sigma_{xx} \) goes to zero, indicating the presence of the high field insulator state.

- Furthermore, a sharp resonance develops, and this resonance increases in frequency with decreasing density.
How do we know this a Wigner Crystal?

• One possibility is that this is the resonance of electrons single-particle localized (trapped) in the impurity potential.

• The shift of frequency of the resonances with carrier density implies that not only the number of oscillating carriers is changing, as would be the case for trapped carriers, but also that the denisty affect the parameters of the oscillation.

• This suggests that electron interactions must play a role in the resonance, and that this is *not* a single-particle effect.

• Can we make any quantitative comparisons of the frequency dependence to models of a Wigner Crystal?
How does a WC pin to Weak disorder?

- Because of disorder, the crystal will be disrupted, and will no longer have long range order.

- If the pinning is weak, the crystal will break up into small “domains”: these domains are regions where the Wigner crystal maintains phase coherence. At the edges of the domains, the phase coherence is lost.

- The “pinning mode” is a shearing oscillation of these domains in the impurity potential.
What is the frequency of the pinning mode?

- For $B=0$, the pinning mode frequency will be that of a transverse phonon of the WC whose wavelength is the same as the domain size $L$:

$$\omega_o = \frac{2\pi c_t}{L}$$

where $c_t$ is the transverse phonon velocity:

$$c_t = \sqrt{\frac{\mu_t}{nm^*}}$$

and $\mu_t$ is the shear modulus. For a classical WC, this is given by:

$$\mu_t = 0.245 \cdot \frac{e^2 n^{3/2}}{4\pi \epsilon_0 \epsilon}$$
What is the frequency of the pinning mode? (cont’d)

- What is the size of the domains? If you minimize the energy of the domain in the weak pinning model, the WC you get:

\[ L = \frac{\mu t a^2}{n_i^{1/2} V_0} \sim n^{1/2} \]

where \( a \) is the lattice spacing, \( V_0 \) is the impurity potential strength, and \( n_i \) is the impurity density. (For detailed calculation, see Fukuyama and Lee, PRL 18 6245 (1978)).

- Note that the domain size depends not only on the impurity strength and density, but also the lattice spacing of the WC.

- Experimentally, domain sizes in the \( \nu = 0 \) magnetically induced WC are range from 250 to 600 nm from the microwave data (PRL 89 176802 (2002)).
What is the frequency of the pinning mode? (cont’d)

• In a magnetic field, the frequency is modified by the cyclotron motion of the electrons:

\[ \omega_{\text{peak}} = \frac{\omega^2}{\omega_c} = \frac{2\pi \mu_t}{nm^* L^2} \]

\[ \mu_t \sim n^{3/2} \text{ and } L^2 \sim n \text{ gives} \]

\[ \omega_{\text{peak}} \sim \frac{1}{n^{1/2}} \]

• Conclusion from FL theory: the peak frequency should increase with decreasing density as \( n^{-1/2} \).
Let's go back to the data...

- Shallow data is fit to a $n^{-1/2}$ power law, and the steep data is fit to $n^{-3/2}$.
- While the $n^{-1/2}$ law is consistent with the predictions from FL theory discussed above, the resonances are much too sharp. (FL predicts $\Delta \omega \sim \omega$).
- More modern theories (for example, Fertig PRB 59 2120 (1999), Fogler and Huse PRB 62 7553 (2000)) show that the long-range Coulomb interaction reduces the inhomogeneous broadening of the pinning mode. They also predict regimes of $n^{-3/2}$ frequency dependence.
Wigner Crystals in the Quantum Hall Effect?

Density is $3 \times 10^{11}$ and mobility is $2.4 \times 10^{7}$. PRL 91 016801 (2003)

- In the data, we see a set of peaks that are shifting higher in frequency as we get closer to a filled landau level, which is consistent with a WC whose density is decreasing.
- But why is there a WC around filled Landau levels?
Electrons in a Nearly Full (or empty) LL

- Electrons in a filled LLs are effectively “inert” because of the cyclotron gap.

- The remaining electrons in the highest LL act like a 2D system whose density is given by $n^* = N \nu^*$, where $N$ is the LL degeneracy and $\nu^*$ is the partial filling factor of the highest LL.

- Just like at low densities in the zero field case, we expect the system to crystallize when $n^*$ is small. (The same holds for when $1 - n^*$ is small, as we can consider this a very dilute gas of holes.)
Global Phase Diagram of the Quantum Hall Effect

From M. Fogler, cond-mat/0111001

- Where are the Skyrmions?
Other Interesting Work done with this Technique

• Measurements of a Magnetically Induced WC near $\nu = 0$ in electrons (PRL 89 176802 (2002)).

• Microwave resonances of the bubble phase crystals of 1/4 and 3/4 filled higher Landau Levels (PRL 89 136804 (2002)).

• Measurements of a Wigner crystal near $\nu = 3$ (cond-mat/0307182).

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