Localization of Fractionally Charged Quasi-Particles


CMX Journal Club Talk
22 February 2005
G. Steele
Outline

1. Single Electron Transistor Charge Sensor

2. Localization in the Integer Quantum Hall Effect
   • “Single particle” vs. self-consistent models
   • Quantum dots/antidots in the low disorder/high field limit

3. Quantum dot localized states in the FQHE
   • Direct measurement of fractionally charged quasi-particles

4. Conclusion

5. Antidot resonant tunneling measurements
   • Why these experiments do not measure the quasi-particle charge
Single Electron Transistor

Gate

Source

Island

Drain

$e^2/C$

N
Single Electron Transistor

Gate

Source

Island

Drain

N
Single Electron Transistor
Single Electron Transistor

Gate

Source

Island

Drain

N
Single Electron Transistor
Single Electron Transistor

Gate

Source

Island

Drain

N+1
Single Electron Transistor

Gate

Source

Island

Drain

N+1
SET as a charge sensor

Typical Charge sensitivity: \( \sim 10^{-5} \text{ el/Hz}^{1/2} \)

(in units of charge induced on the gate)
Scanning SET

SET fabricated on the end of a pulled glass fiber
“Transparency” Measurements

SET measures electric field penetrating the 2DES due to the finite density of states.

SET is small, and therefore sensitive to local charge fluctuations in the 2DES.
The Quantum Hall Effect

Fig. 1. Recording of $V_H$ and $V_X$ versus magnetic field for a GaAs device cooled to 1.2 K. The current is 25.5 $\mu$A.
The Quantum Hall Effect (cont’d)

Laughlin: If $\sigma_{xx}$ of the bulk goes to zero, then the Hall resistance is exactly quantized.
SET Transparency Measurements in the IQHE

Density is varied by applying DC voltage to the backgate.

There is a filled Landau level whenever:

\[ n = (eB / h) * i \]

where \( i \) is an integer, producing a “fan” pattern in the B-n plot.
Why do we see sharp fluctuations around filled Landau Levels?

SET shows a spike in the AC signal each time a quantized state moves through the Fermi energy

Electrons localized by a radial electric field from a dip in the potential

Each state encloses an integer number of flux quanta h/e
Take a closer look at the fluctuations

$\hbar \omega_c - E_F = \text{Constant}$

Lines running parallel to constant filling factor

Area enclosed by ring is constant

Chklovskii cond-mat/9609023
Cobden PRL 82 4695 (1999)
But wait a minute…

Each single particle state encloses exactly one more flux quantum than the previous one. Increasing the field while keeping the area constant should increase the number states inside the ring!

*But the number of charges is seen to stay constant!*

---

**Graph:**
- **Density** vs. **Magnetic Field**
- Four states labeled 0 to 4.
- Two sets of data points for different magnetic fields:
  - $B = B_0 \quad v = 0.7$
  - $B = 2B_0 \quad v = 0.7$
What is wrong?

In the “single particle” picture, we draw Landau levels that follow the zero field potential profile.

However, this leaves behind uncompensated charge!

Resulting electric fields from the uncompensated charge try to empty filled states and fill empty ones.
In self consistent solution, the density must follow closely the zero field density to avoid large amounts of uncompensated charge.

However, each orbit center can only hold zero or one electron!

The only way to avoid uncompensated charge is if the self-consistent electrostatic potential has flat regions, making states at different orbit center positions degenerate in energy.

Once the states are degenerate, we can reproduce the zero field density profile by partially occupying the multiple degenerate states.

Chklovskii, Shklovskii, Glazman \textit{PRB} 46 4026 (1992)
(McEuen et. al. \textit{PRB} 45 11419 (1992) for quantum dots)
Localized states with self-consistent screening

Number of charges = Number of states

Number of charges determined by area of “bubble” above filled LL independent of number of states!
Localized states with self-consistent screening

Discrete charging peaks due to quantized single particle energy spectrum

Number of charges = Number of states

Number of charges determined by area of “bubble” above filled LL independent of number of states!
T-F Calculation for realistic disorder potential

Quantum antidot

Quantum dot
Scanning SET
Width of the localized regions
Width of the localized regions

\[ n \left(10^{10} \text{ cm}^{-2} \right) \]

\[ B (\text{T}) \]

\[ \Delta n \]

\[ \nu = 1 \]
Width of the localized regions

\[ n (10^{10} \text{ cm}^{-2}) \]

\[ B (\text{T}) \]

\[ \Delta n \]

\[ \nu = 1 \]
Width of the localized regions

![Diagram showing width of localized regions](image)
Width of the localized regions

\( \nu = 2 \)

\( \nu = 1 \)
Width of the localized regions

\[ v=2 \]

\[ v=1 \]
Width of the localized regions

\[ n \left(10^{10} \text{ cm}^{-2}\right) \]

\[ B (\text{T}) \]

\[ \Delta n \]

\[ v=1 \]

\[ v=2 \]
Width of the localized regions
Width of the localized regions

The self consistent model also accounts for the fixed density width observed when $\Delta n < n_{LL}$ (the high field/low disorder limit)
Low field / high disorder limit

For $\Delta n_{\text{dis}}>n_{\text{LL}}$, the phenomenology of SP model is reproduced, with metallic strips following trajectories of SP localized edge states.
What about the Fractional Quantum Hall Effect?

FQHE shows same microscopic localization mechanism as IQHE in high field/low disorder limit

CB Period tripling?
Scanning SET: Fractional Coulomb Blockade

Scanning SET results conclusively demonstrate the fractional quasi-particle charge.
Conclusions

1. SET electrometer measurements confirm the self-consistent potential model of localization

2. In the high field/low disorder limit ($\Delta n > n_{LL}$), localization is driven by confinement of quantized quasi-particle charge onto small islands created from degenerate single particle orbitals

3. Fractional quasi-particles are localized in the exact same way, and the charge of the $1/3$ and $2/3$ quasi-particles are directly measured to be $e^* = e/3$
<table>
<thead>
<tr>
<th>“Single Particle” (self-inconsistent) Model</th>
<th>Self Consistent Model (including “interactions” at the Hartree level)</th>
</tr>
</thead>
<tbody>
<tr>
<td>• <em>Potential</em> profile is assumed to be the same as at zero field</td>
<td>• <em>Density</em> profile is seen to follow the profile from zero field, with narrow modifications (“incompressible strips”) at integer filling</td>
</tr>
<tr>
<td>• All states are either empty or full</td>
<td>• States in compressible regions are made degenerate by the self consistent potential and then partially occupied so as to be able to screen uncompensated charge. The is required because of the energy gap.</td>
</tr>
<tr>
<td>• “Edge states” are infinitesimally narrow, separated by wide incompressible regions</td>
<td>• Compressible strips at edges are wide, separated by narrow incompressible strips</td>
</tr>
</tbody>
</table>
Quantum Antidot Resonant Tunneling

Scattering from one edge state to the other through the confined edge state circling around the “antidot”.

Diagram showing the quantum antidot resonant tunneling with arrows indicating the flow of the edge states.
Quantum Antidot Resonant Tunneling

- Assume one state is added to center of “antidot” per flux quantum: use B-dependence to calibrate the radius of “antidot”:
  \[ \Delta B = S_m \phi_0 \]

- Now deplete sample very slightly using a distant backgate. States are moved from the outside of the ring to the inside with a period:
  \[ \Delta V_{BG} = q/C_{BG} = q d_{BG}/\varepsilon_0 S_m \]

- Solve for q:
  \[ q = \varepsilon_0 \phi_0 \Delta V_{BG} / d_{BG} \Delta B \]
Wait a minute: didn’t we just learn that for the self consistent antidot, we don’t add a charge for each flux quantum?

Antidot “disc” vs. Antidot “ring”:

Yacoby antidot
\( (\Delta n < n_{LL}) \)

Goldman antidot
\( (\Delta n > n_{LL}) \)

Must add another empty state for each flux through ring
Now we always have a partially filled state at $E_F$: what prevents us from tunneling all the time?

Answer: Coulomb blockade of the compressible ring.

Even more subtle note

In the SP-like limit, where $\Delta n > n_{\text{LL}}$, changing backgate voltage shifts states into the central incompressible region from outside that region.

In the backgate measurements, it has only been established that in moving 1e of charge from outside to inside the ring, 3 states had passed through the Fermi level: it does not count the number of particles you have moved through the ring, and thus is insensitive to the quasi-particle charge.